

Statistical Modeling of Crack Growth and Reliability Assessment of High-Density Polyethylene

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In this work, a statistical evaluation of the crack-growth process in high-density polyethylene (HDPE) was carried out. The specimens were compression molded from virgin, molding-grade HDPE. Edge-notched specimens for replicate fatigue testing were prepared from compression-molded sheets. Fatigue test results were then analyzed, and it is shown that if the crack-growth process can be characterized as a random process following a power-law-type behavior, then the time to reach a critical crack length will be distributed according to an inverted lognormal model.

Keywords crack growth, fatigue, high-density polyethylene, reliability modeling

1. Introduction

Over the last few decades, solid polymers have become a popular replacement for conventional metallic materials in load-bearing engineering applications. Their light weight, ease of manufacture, corrosion resistance, and relatively lower cost are desirable properties that have led to their popularity. A number of advanced engineering plastics such as polyphenylene oxide (PPO), polyamideimide (PAI), polyetheretherketone (PEEK), and so forth have been developed for certain engineering applications where an excellent mechanical and chemical property combination is required. These engineering polymers provide much better property combinations at a relatively higher cost than their more common counterparts. The conventional less-expensive plastics remain a material of choice for the less-critical applications. The bulk of plastic production still leans toward the conventional, commodity plastics such as polyethylenes, polypropylenes, polystyrene, and polyvinylchloride. Some of these polymers are being used in light-load-bearing applications, such as automotive trim components, hinges and fasteners, small gear and pinion assemblies, worm and worm wheel parts, and so forth. These components are often subjected to cyclic loading, although subjected to light loads, due to vibration or other effects induced during their service life. It is also well known that materials subjected to cyclic loading fail at a stress level much lower than the tensile strength of the material. Plastics are no exception. However, extensive studies on the dynamic fatigue behavior of solid polymers have not been reported in literature.

ElHakeem and Culver (Ref 1) conducted experiments on the fatigue behavior of high-density polyethylene (HDPE). They studied the fatigue-crack propagation (FCP) under different environmental conditions. An empirical model was developed to take into account the test frequency, as well as the level and amplitude of the stress-intensity factor. The crack-opening displacement and the plastic zone size were also taken into account in the formulation of this model. Bucknall and Dumpleton (Ref 2) carried out experiments on HDPE to investigate the FCP under load control. The limited applicability of the Paris equation ($dl/dN = C(\Delta K)^n$) for the solid polymers was indicated, and the crack-growth dependence on loading history was demonstrated. In a subsequent work (Ref 3), these workers conducted a more detailed study of the effects of previous loading history on FCP of three different plastics. Mandell et al. (Ref 4) have also attempted to study the fatigue resistance of two kinds of plastic materials.

All of these studies have concentrated on more classic aspects of FCP in solid polymers such as effects of level and amplitude of stress, environmental effects, loading history, crack blunting, morphology of crack surface, and so forth. Although it is well established that, in general, the fatigue behavior of materials is a probabilistic phenomenon, a few, if any, studies

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Nomenclature

l	Crack size, mm
N	Number of cycles
C	Paris power law constant
ΔK	Stress intensity factor range, MPa$\sqrt{m}$
n	Paris power law exponent
α	Crack length at N = 1, mm (random variable)
β	Power of crack growth equation, (random variable)
$\Phi(N,)$	Normal cumulative distribution function of random variable N
A	Slope of Φ^{-1} (Ni, μ_l, σ_l) versus ln$N$ line, where Φ^{-1}(.) is inverse normal function
B	Intercept of Φ^{-1}(Ni, μ_l, σ_l) versus ln$N$ line
σ_{min}	Minimum applied stress, MPa
σ_{max}	Maximum applied stress, MPa
μ(.)	Mean of the quantity (.)
σ(.)	Standard deviation of the quantity (.)
$F(N)$	$P[N \le N]$, Cumulative distribution function of number of cycles N

have been carried out in this regard. This behavior should be more pronounced for the plastics, owing to the variability due to their molecular-weight distribution, effect of processing parameters, anisotropy introduced due to molecular-chain alignment, presence of crystalline and amorphous regions, and so forth. In the present study, an effort will be made to investigate the variability of fatigue properties of the plastic materials and a probabilistic representation of the crack-growth process. The model material used in the study was molding-grade HDPE produced by Saudi Basic Industries Corporation (SABIC). The specimens were compression molded in the form of 6 mm thick sheet. The compression molding was done at 170 °C. The mold was first cooled at 2 °C/min to 100 °C and then at a rate of 1 °C/min to room temperature.

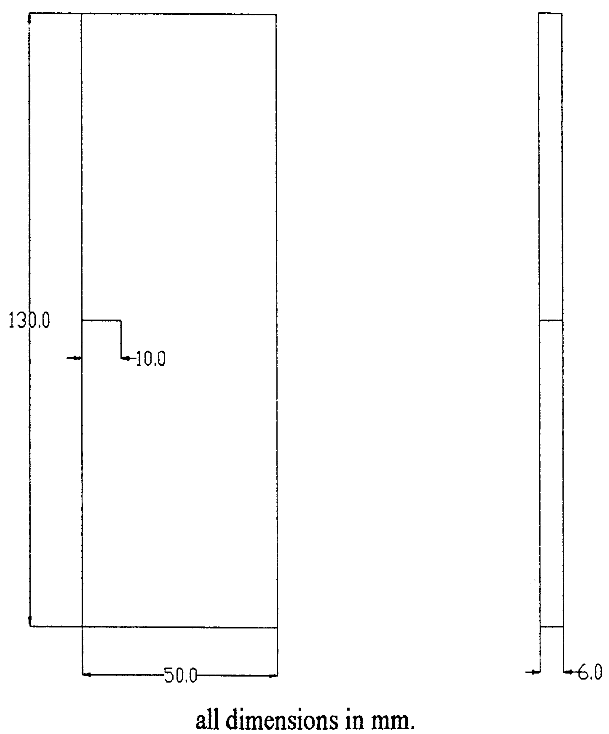

Fig. 1 Fatigue-crack-growth specimen geometry of HDPE

2. Fatigue Testing

Replicate fatigue tests were carried out using the single-edge-notched rectangular specimen, the dimensions of which are shown in Fig. 1. The notch was introduced by pressing a paper-cutting razor blade into the edge of the specimen, which was held in a fixture to ensure a constant depth in all the test coupons. All tests were conducted under identical test conditions of ambient laboratory environment and testing parameters. A servocontrolled electrohydraulic materials testing system was used for the fatigue testing. Fatigue cycling was carried out under sinusoidal loading at a frequency of 1 to 5 Hz and a stress ratio of $R = 0.1$ ($R = \sigma_{min}/\sigma_{max}$). Crack-growth rates were monitored and measured optically by using the Questar QM-100 (Questar Corporation, New Hope, PA) long-distance traveling microscope. This microscope provided precise measurement of the crack length on a digital readout unit with an accuracy of 0.01 mm. All crack-growth measurements were carried out using a magnification of 40×. To ensure identical initial conditions in all test coupons, a fatigue starter crack of 2 mm length as generated from the notch by fatigue testing at a higher load amplitude and a higher frequency of 20 Hz. All

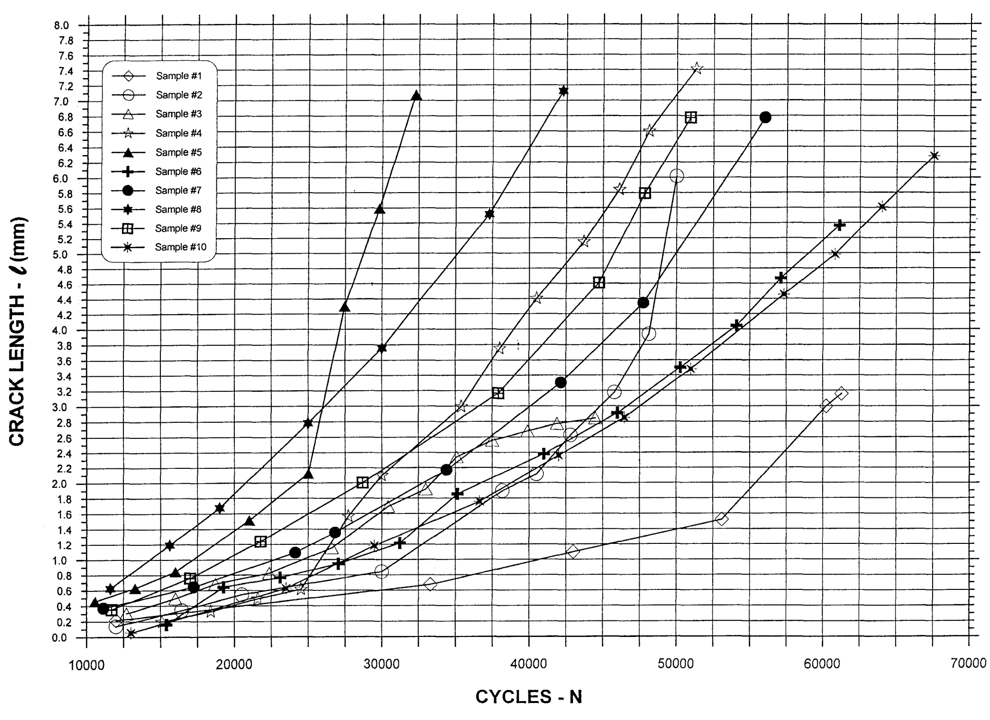

Fig. 2 Statistical process of crack growth. Crack length versus cycles for HDPE

crack-growth rate measurements were carried out taking this starter crack as the initial crack length.

3. Fatigue-Crack-Growth Rate Results

The results of the replicate fatigue-crack-growth rate tests for the HDPE plate specimens are presented in Table 1. The same results are graphically shown in Fig. 2 as crack length versus number of cycles. As can be noticed from Fig. 2, a considerable scatter is observed in the crack-growth data. This scatter tends to increase with the number of fatigue cycles, or in other words the scatter increases with increase in the crack length. For example, at N = 20,000 cycles the crack length in the replicate tests varies from 0.3 to 1.8 mm. At N = 30,000 cycles, the crack length is distributed between 1.0 and about 5 to 6 mm.

4. Data Analysis

Figure 2 illustrates that the crack-growth process in HDPE is essentially a statistical phenomenon. Similar sample func-

Table 1 Crack-growth data of HDPE

Sample 1		Sample 2		Sample 3		Sample 4		Sample 5	
l, mm	*N*	*l*, mm	*N*	*l*, mm	*N*	*l*, mm	*N*	*l*, mm	*N*
0.215	12,000	0.137	12,000	0.301	12,700	0.197	15,100	0.458	10,580
0.671	33,300	0.341	16,440	0.494	16,000	0.326	18,400	0.629	13,270
1.097	43,000	0.543	20,500	0.682	18,700	0.489	21,570	0.841	16,000
1.519	53,100	0.843	30,000	0.822	22,350	0.610	24,500	1.512	21,000
2.995	60,230	1.894	38,200	1.152	26,600	1.563	27,710	2.119	25,000
3.155	61,280	2.114	40,500	1.692	30,420	2.096	30,000	4.298	27,450
3.296	62,777	2.626	42,820	1.914	33,000	2.990	35,400	5.592	29,800
3.690	63,826	3.174	45,800	2.337	35,070	3.750	38,000	7.074	32,250
4.043	63,184	3.936	48,120	2.554	37,500	4.402	40,500	9.903	34,022
4.619	66,384	6.013	50,000	2.676	39,900	5.148	43,700		
5.204	67,270	10.104	51,500	2.773	41,900	5.840	46,100		
6.671	68,970			2.844	44,500	6.599	48,100		
7.332	70,150			8.906	48,700	7.411	51,300		
8.307	71,070			9.215	50,000	8.206	54,010		
9.596	71,430			9.618	51,500	9.375	57,440		
				9.984	53,000	10.066	59,050		
				10.536	54,500	10.848	61,000		
				11.058	55,800	11.414	62,500		
				11.745	56,700	12.001	64,020		
				12.315	58,000	12.689	65,540		
				13.003	59,005	13.329	66,920		
				13.658	60,090	14.294	68,300		
				14.340	61,490	15.013	69,170		
				14.890	62,300				

Sample 6		Sample 7		Sample 8		Sample 9		Sample 10	
l, mm	*N*	*l*, mm	*N*	*l*, mm	*N*	*l*, mm	*N*	*l*, mm	*N*
0.159	15,400	0.371	11,150	0.624	11,600	0.353	11,700	0.056	13,000
0.641	19,250	0.645	17,240	1.191	15,625	0.760	17,000	0.623	23,500
0.766	23,100	1.089	24,120	1.674	19,000	1.238	21,775	1.181	29,500
0.944	27,040	1.354	26,810	2.782	25,000	2.002	28,700	1.756	36,650
1.215	31,230	2.166	34,400	3.756	30,000	3.166	37.,920	2.358	42,000
1.854	35,150	3.304	42,180	5.515	37,280	4.609	44,720	2.853	46,485
2.378	41,000	4.343	47,750	7.122	42,250	5.790	47,830	3.481	51,000
2.909	46,000	6.772	55,990	8.791	46,460	6.774	50,930	4.455	57,330
3.501	50,300	7.358	58,000	9.640	48,500	7.831	53,930	4.982	60,800
4.044	54,100	8.386	60,160	10.348	50,000	9.588	56,870	5.612	64,000
4.669	57,100	8.929	62,180	11.016	51,050	10.623	58,700	6.281	67,500
5.364	61,100	10.051	64,240	11.912	52,000	11.865	60,280	6.708	70,000
6.798	65,100	11.209	66,350	12.621	53,040	12.686	60,440	7.355	72,800
8.059	68,050	12.049	68,250	13.362	53,820	13.438	60,510	8.142	75,130
9.459	71,200	13.225	70,250			14.598	60,550	8.523	77,090
11.214	74,150	13.866	70,970					9.692	79,500
12.502	76,500	14.600	71,790					10.528	82,630
13.827	78,050	16.038	72,205					11.088	83,500
14.346	78,670							11.786	84,320
14.745	79,010							12.788	84,840
15.251	79,470							13.788	84,940
16.106	79,580							15.018	85,000

l, crack length; *N*, total number of cycles

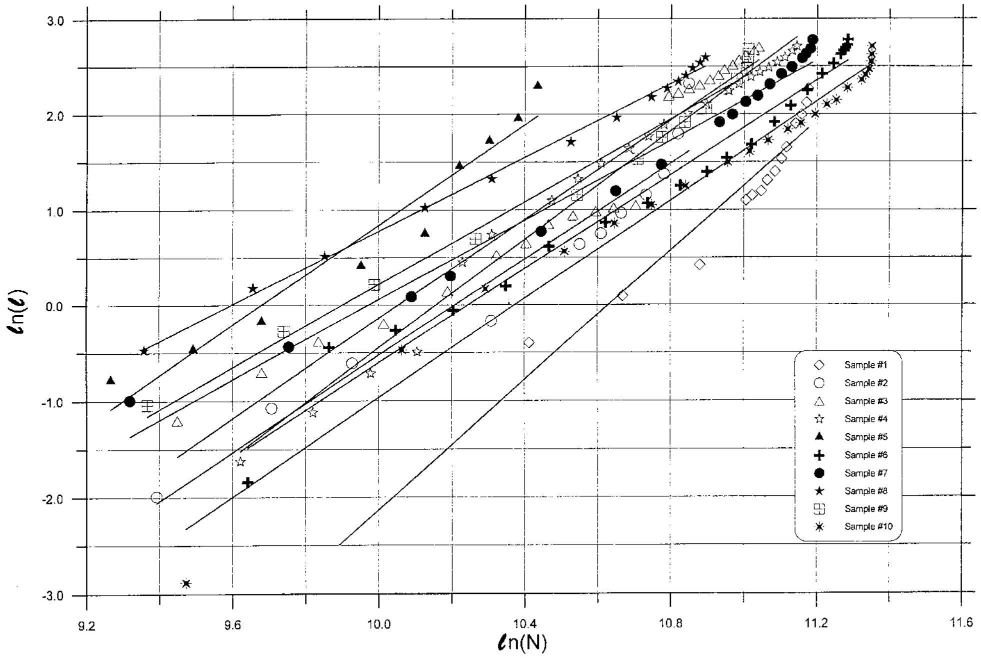

Fig. 3 ln (l) versus ln (N) plot of realizations crack-growth process

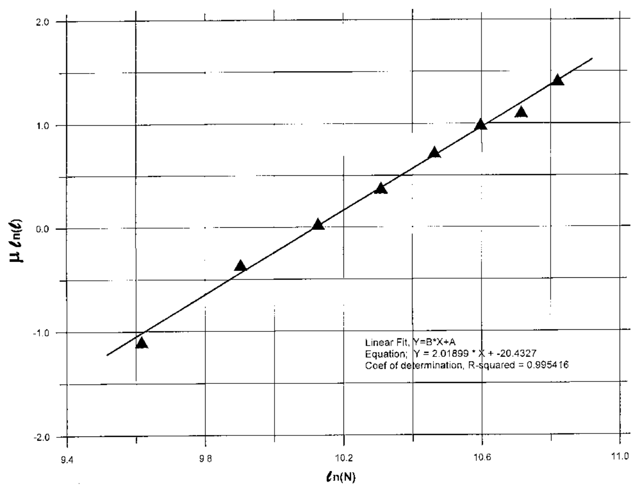

Fig. 4 μ ln (l) versus ln (N) plot

tions of crack growth in metallic alloys have been observed, and a variety of approaches and models have been used to analyze such data (Ref 4). For this crack-propagation data, a rather simple mathematical approach as outlined in a recent paper by Sheikh and Younas (Ref 5) is used. This approach of reliability modeling is based on the following characterization of crack growth and underlying assumptions:

Assumption 1:

$$l = \alpha N^{\beta} \qquad \text{(Eq 1)}$$

$$\ln l = \ln \alpha + \beta \ln N \qquad \text{(Eq 1a)}$$

which indicates that the crack-growth process is a power-law random process with highly correlated increments. Figure 2 represents a set of sample functions of this process. Each individual curve (or sample) function of Fig. 2 is fitted to $\ln l_i = \ln \alpha_i + \beta_i \ln N$, in Fig. 3 for $i = 1, 2, 3, \ldots$, using regression approach; high R^2 values have been observed in each case, which demonstrates that Eq 1 or 1(a) can be assumed to represent the crack-growth process of Fig. 2.

Assumption 2: From Eq 1(a), the mean of $\ln l = \mu(\ln l)$ and standard deviation of $\ln l = \sigma(\ln l)$ can be established as follows:

$$\mu(\ln l) = \mu(\ln(\alpha)) + \mu(\beta) \ln(N) \qquad \text{(Eq 2)}$$

and

$$\sigma(\ln l) = [\sigma^2 (\ln l) + \sigma^2(\beta) \ln N^2]^{1/2} \qquad \text{(Eq 3)}$$

where it is assumed that α and β are independently distributed random variables. Figures 3 and 4 indicate the validity of Eq 2 and 3 for the data presented in Fig. 2. Figures 3 and 4 demonstrate that both the average of (ln (crack size)) and standard deviation of ln (crack size) are linearly varying with respect to ln N, and their ratio or coefficient of variation of ln l is independent of time (Fig. 5).

Assumption 3: It is also important to characterize the evolution of the distribution of crack size as a function of time. At various time locations $N_1, N_2, \ldots$, the distribution of crack size $f(l(N_1)), f(l(N_2)), \ldots$, are plotted in Fig. 6. The vertical axis is $F^{-1}(N_i, \mu_i, \sigma_i) = A \ln N + B$ when $F^{-1}(N_i, \mu_i, \sigma_i) = \Phi^{-1}(i/(N+1))$. Figure 6 clearly demonstrates that the quantity ln l is normally distributed at different ln N. In other words, l is lognormally distributed at time ln N. Similarly, slopes of ln l versus ln N curves are plotted in Fig. 7, which demonstrates that their slopes are also normally distributed.

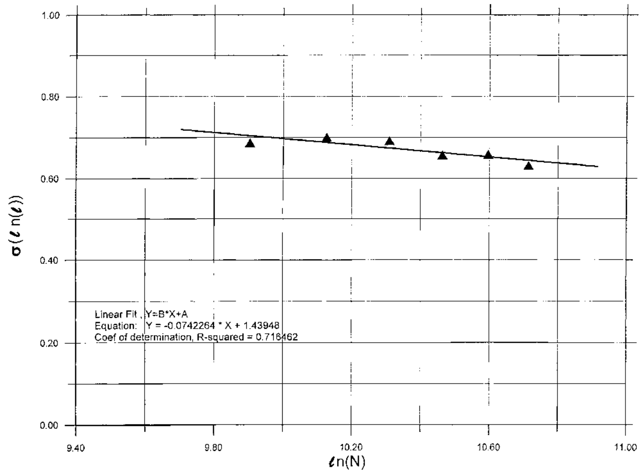

Fig. 5 σ (ln (*l*)) versus ln (*N*) plot

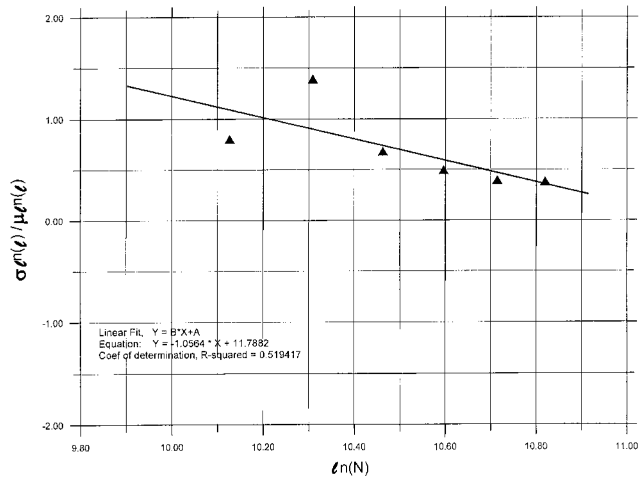

Fig. 6 σ ln (*l*)/μ ln (*l*) versus ln (*N*) plot

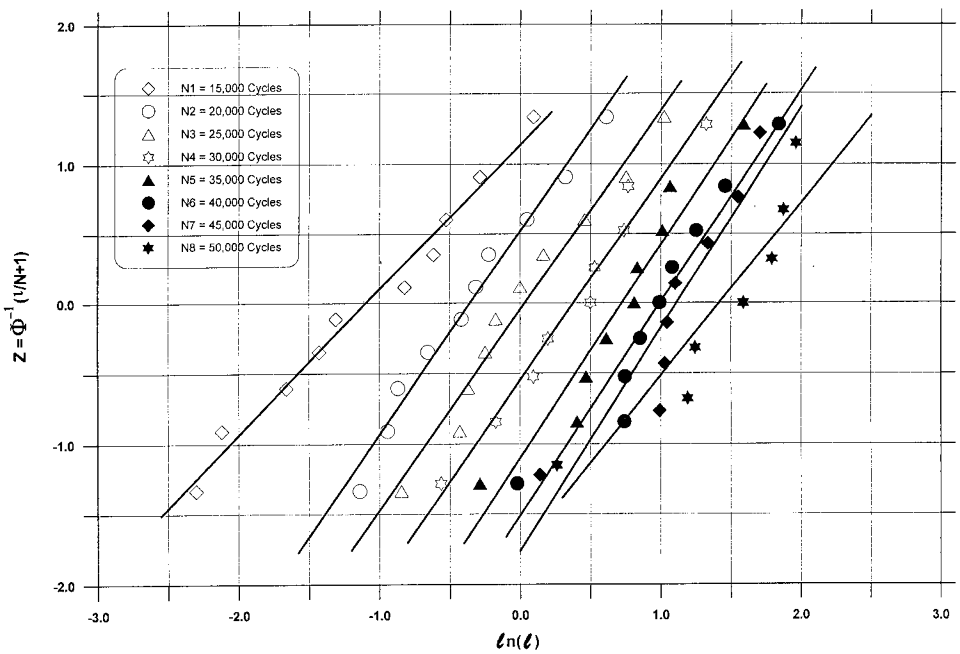

Fig. 7 Normal distribution of ln (*l*) at various time locations

Thus, all the underlying assumptions of the model proposed by Sheikh and Younas (Ref 5) are fully applicable in this case. Based on these observations and definition of life as $\ln N = (\ln l_c - \ln \alpha)/\beta$, where l_c is the critical level of crack size that can be tolerated using the equivalency of the probabilistic events $P[l(t) \leq l_c] = P[N > N]$, the reliability of the polymer under consideration at any time N will be:

$$R(N) = \Phi[(1 - \ln (N_m/\ln N)/\sqrt{A_l}] \qquad \text{(Eq 4)}$$

The parameters of the distribution $R(N)$ can be determined by using linear regression to transformed data as outlined by Sheikh and Younas (Ref 5). The regression lines to the transformed data are given in Fig. 8 at three different arbitrarily selected critical levels of crack size, which indicate that an inverted lognormal model can very well characterize these data. The parameter A_l represents the scatter in life at specified levels of critical crack size. This parameter is related to the scatter of crack growth as follows (Ref 5):

$$A_l = \sigma^2(\beta)/\mu^2(\beta) \qquad \text{(Eq 5)}$$

The other parameter $C_l = \ln N_m$ is related to the median time to reach critical crack size. Fifty percent of the crack-growth curves would have reached l_c by this time, and that is related to the average crack-growth characteristics and rate of its propagation as follows:

$$C_l = (\ln(l_c) - \ln(\alpha))/\mu(\beta) \qquad \text{(Eq 6)}$$

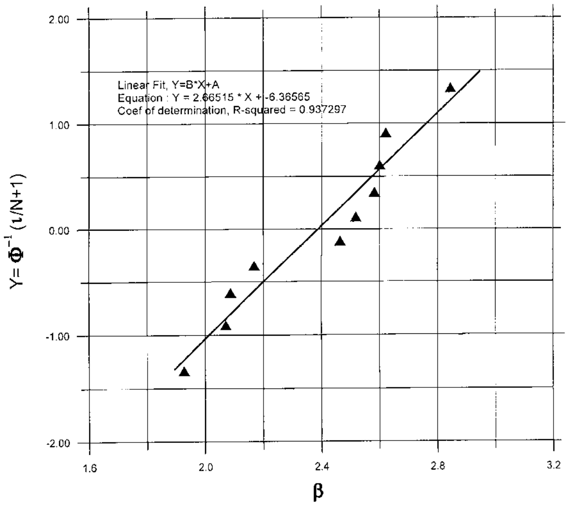

Fig. 8 Normal distribution of crack-growth parameter beta

Figure 9 illustrates fitted curves using parameters A_l and C_l estimated by regression. Fitted distribution represented by nonparametric distribution indicated by continuous curves are superimposed on the nonparametric distribution.

5. Discussion and Concluding Remarks

In polymeric materials, if crack growth can be modeled as a random process characterized by $l(N) = \alpha N^{\beta}$ as in the case of HDPE, then the time to reach a critical (or threshold) value of crack size will be distributed according to an inverted lognormal model. The parameters of this model can be directly determined from crack-growth process parameters. The reliable life corresponding to specified state of failure p can be determined as:

$$N_p = \exp [C_l/(1 - \sqrt{A_l}\, Z_p)] \quad \text{(Eq 7)}$$

where Z_p is the solution of $\Phi(Z_p) = p$. These reliability models can be integrated in a comparative assessment of the products made by HDPE, part-replacement strategies, and damage-tolerance design with polymeric materials. More experimental

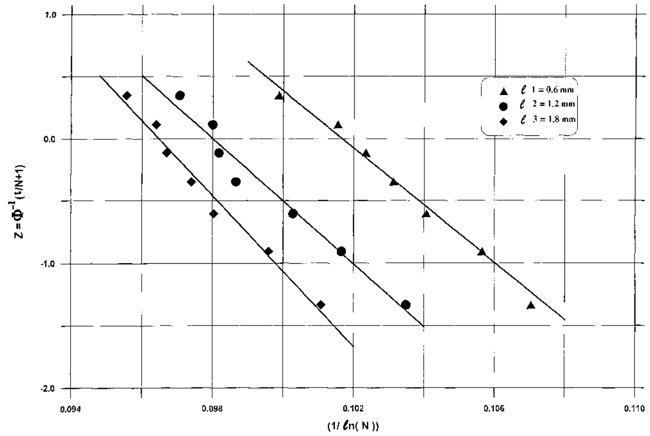

Fig. 9 Inverted normal distribution fit, time to reach a critical length of crack l

studies are needed to integrate it with Paris-law-type characterizations of the crack-growth process.

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